

# 10. Man-Machine Models for Car Steering\*

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The driver steering task and its environment are briefly described and the known properties of driver, highway, and vehicle reviewed. A three-level, information-processing model for driver steering control is developed.

Level 1 uses preview estimates of highway curvature as a basis for open-loop adjustment of vehicle path curvature. Level 2 uses relative heading, relative path-angle, or lateral position-rate to null out residual lateral position-rate errors. Level 3 uses lateral position itself in a low-frequency, low-gain loop to remove residual error. Evidence for each of these is briefly reviewed.

An auxiliary routine for controlling vehicle velocity to meet constraints on lateral acceleration is proposed as an adjunct to the model.

## INTRODUCTION

This paper discusses the structure of what is almost certainly the most common manual control task in modern life, viz. car steering. Back-of-the-envelope calculations indicate that some  $10^{10}$  to  $10^{11}$  manhours are spent per annum driving automobiles; a major part of this task is keeping them in lane while traveling on highways, maneuvering them in and out of parking lots, etc. Since the penalties for failure are high including damage to property, personal injury and lost time, extreme reliability is demanded of the man-machine system. It is in general achieved, as the low accident-rates per vehicle mile indicate. This fairly satisfactory position appears to have been reached by evolutionary improvement in vehicle and highway design and maintenance, aided by the human driver's ready acquisition of control skills and his great adaptive powers, rather than by an explicit analytical design procedure, and in this respect car driving resembles most other manual control systems.

The residual failure rate which is currently a matter of public concern, must be associated with faults or breakdowns in one or more of the three components: driver, vehicle, and highway. The fact that there are numerous distinct types of failure, each making a small contribution to the overall rate, indicates that the evolutionary process must have reached a steady state or plateau. Unless revolutionary changes such as automatic

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steering are introduced, further progress can only be sought through attention to numerous relatively small details and specific features of the man-machine system, each by itself producing only a small increment of performance and reliability. To facilitate this process, a fully analytical and experimentally substantiated theory of the highway/driver/vehicle control system is required. The present paper sets out our present thinking on the form such a theory should take.

The general outline of a car steering theory has been developed by several previous authors, but at present there appears to be no general agreement either on exact topology or on quantitative estimates of the various parameters involved. Rashevsky (ref. 1) and Wohl (ref. 2) pointed out the relatively simple closed-loop structure of the steering task but did not give adequately detailed treatments nor cite experimental evidence in support of their analyses. Weir (ref. 3) has reviewed the possible loop-closures and performance likely to be obtained from them, but does not take preview and open-loop response modes into consideration. Perhaps the most explicit frequency domain model of the driver is that recently proposed by Wierwille et al. (ref. 4) derived from experimental model-matching studies of three subjects steering a winding-highway task at a fixed 68.2 mph under full (450 ft) and zero preview in a fixed base simulator with pure second-order vehicle dynamics ( $0.185/s^2$ ). The model developed for the former condition represents the driver's output as the sum of a feedforward (open-loop) transfer from highway lateral displacement (a polynomial in the Laplace-transform variable  $s$ ), and a feedback (closed-loop) transfer from vehicle lateral position error, referred to an arbitrary zero. While feedforward control was introduced for the first time, no attempt was made to separate the various feedforward and feedback paths that could in principle be employed by the driver using the different channels of visual, kinaesthetic, and vestibular information available to him, nor was the effect of forward speed on vehicle dynamics taken into account.

In the present paper we first discuss the known properties of the three major system components—highway, driver, and car—in a generalized way, and then present an outline model of the car/driver/highway interaction based on these and on certain of our recent experimental results. No attempt is made to develop a full analytical treatment, partly for reasons of space and partly because we feel that at the present stage discussion should be focused on the macrostructure and topology of the system before getting down to specifics of the analytical formulation. Following this line of thought we have mixed digital-type information processing models rather freely with continuous-control models, choosing whichever seems most suitable at each point. Once precisely formulated, each of these may be rewritten in terms of the other to yield a consistent overall treatment. But we do not yet know which is more useful.

## ENVIRONMENT AND TASK DESCRIPTION

Consider the general cross-country driving situation. This includes both straight and winding roads, with and without traffic. Speeds range between zero and, say, 75 mph, and may at times fluctuate rapidly. The pavement may be level, tilted, or uneven, and there will be side-forces due to wind, both natural and that caused by other vehicles. The visual field will generally be rich in detail and structure; forward visibility is normally several hundred feet, but may be restricted by fixed obstructions, by other traffic, by darkness, and by atmospheric conditions such as rain, fog, and snow. In the latter case we may assume a 200-foot minimum for normal driving.

The steering task varies from moment to moment when passing and fixed obstacles are considered, but for a large part of the time the driver must stay within his traffic lane despite disturbances. With little loss of generality we may assume the lane defining the

lateral position tolerance to be 12 feet wide, demarcated by white painted lines on either side. If the car is 6 feet wide there will be no penalty associated with fluctuations in lateral position up to  $\pm 3$  feet from the center of this lane, but in certain frequent circumstances (presence of fixed obstacles and other traffic) the "cost" of errors exceeding this increases steeply.

The control system currently utilized may conveniently be split into three components: driver, vehicle, and highway geometry (fig. 1). The latter takes into account the consequences of forward motion and of the driver's distant vision. Since the properties of highway and vehicle and the geometrical relationships are all well understood, our current research problem is to model the driver adequately enough to account for the known facts of steering performance, and to permit predictions under variation of design parameters.

### STEERING SYSTEM COMPONENTS

THE DRIVER. — Certain general remarks can be made about the driver's information processing capability which limit the scope of possible driver models.

(1) With his remarkable pattern recognition powers, the driver may be assumed capable of acquiring any desired signal or data present in his visual field, filtering and correcting distortion. However, the precision of any quantitative estimate cannot exceed around 3 bits per dimension per observation (ref. 5).

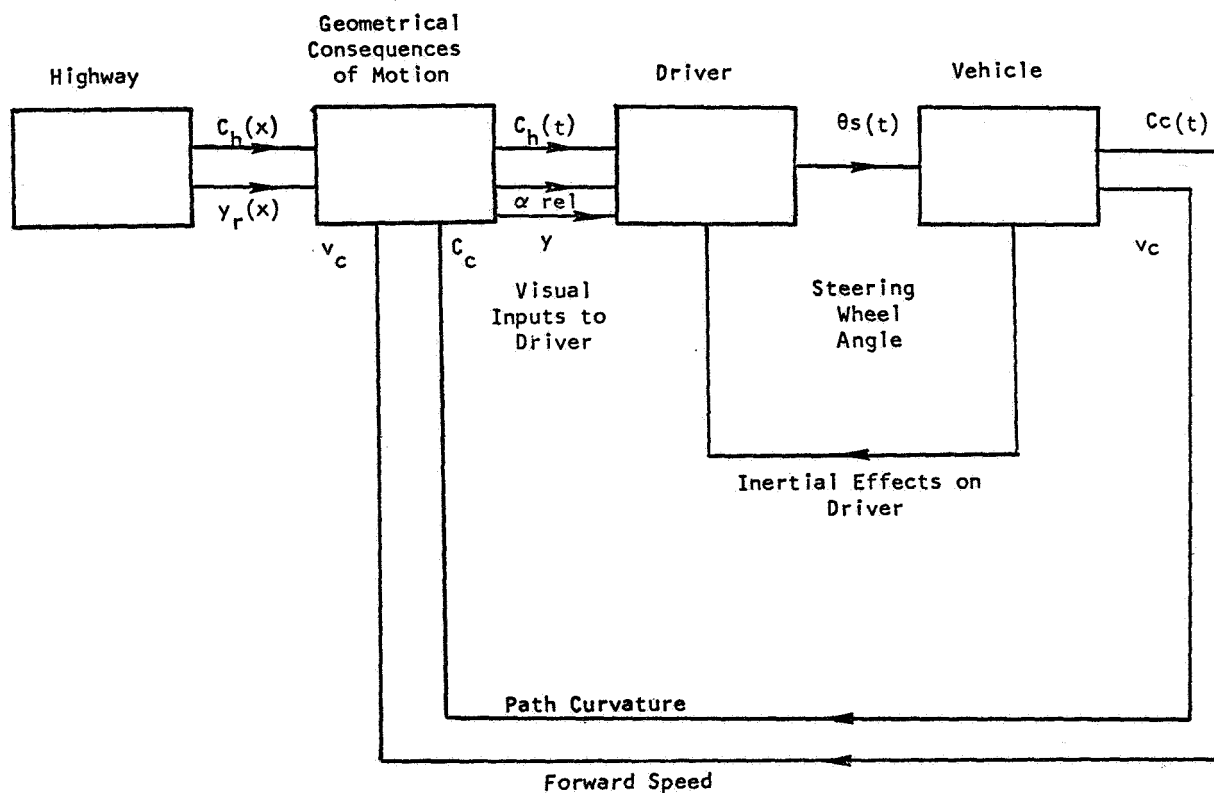


Figure 1. —General layout of the steering subsystem.

(2) The selective power of the driver's eye-fixation system and brain permits independent data to be acquired from visual fields as small as  $3 \times 10^{-4}$  or as large as  $3 \times 10^{-2}$  steradians ( $1^\circ$ - $10^\circ$  subtense). However shifts of attention take significant time so that frequent switching is uneconomical and attention is likely to be focused on only one area of the visual field at a time.

(3) The driver may take the first time derivative of current input signals, but this entails a marked loss of precision, and, it is safe to assume, that only 1 to 2 bits of rate information are available for any observed variable.

(4) Judgments of relative time are also subject to about a 3-bit limitation so that precise purposeful time delays or prediction cannot be assumed.

(5) Inputs from vestibular senses and body musculature provide both linear and angular acceleration data for, respectively, the head and the seat of the pants, but again with strictly limited precision (ref. 6).

(6) The effector subsystem, normally two hands applying torque to the steering wheel, can set an angular position or apply a desired torque with strictly limited precision (about 2-1/2 bits); it can hold an existing position against changes of torque; and produce a desired angular rate or torque rate with somewhat lower, perhaps 2-bit, precision.

(7) The effector subsystem may also supply data ("feel") on the torque currently being applied and on high-frequency load-changes or torque inputs.

(8) There is no evidence that central processes can operate at any higher precision than the input and output channels described above.

(9) There is a strong likelihood that central computations and decisions cannot be multiprocessed, hence that sequential programing must be assumed. However input and output buffers permit some flexibility of central process timing.

(10) Successful task performance normally requires a prolonged period of skill acquisition, hence we may assume complex information processing programs and the presence of much data in permanent storage.

Considering these constraints, the natural form of driver model would seem to be a sequential information processor like a digital computer, but with severely limited transient data storage, word-length 2 or 3 bits, and a small instruction set. Long sequences of processes would be substituted for precision. This viewpoint is supported by recent studies on principles of direct digital control (ref. 7).

If a quasi-linear continuous control model is adopted, the generally low precision of the driver's input data-channels and output adjustments and sequential computation capacity suggests that it must assume low-order dynamics except at frequencies directly involving neuromuscular dynamics, which are not generally of interest in car steering. We must also expect low correlation (coherency) between input and output.<sup>1</sup> Since noise is introduced in processing, we must expect the driver to combine data obtained through two or more channels to provide enough information to fully utilize his output capacity.

**THE HIGHWAY.** — The desired path of the vehicle along the highway, which is the primary disturbance or forcing-function input to the vehicle-driver system, can be best specified as a time-invariant space function  $\alpha_h(\underline{x}_h)$  where

$\alpha_h$  = tangent to the direction of the highway centerline

$\underline{x}_h$  = arc-length position along the highway

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<sup>1</sup>However if all the information in a single 3-bit input were reproduced at the output the correlation obtained would be  $r=0.92$ , and even 1-bit transfer yields  $r=0.77$ .

Since highways are generally constructed out of straight sections connected by circular arcs, the function  $\alpha_h(x_h)$  is a special kind of stochastic variable, whose first derivative, the highway curvature  $c_h = d\alpha_h/dx_h$  takes a sequence of randomly chosen constant values for successive sets of values of  $x_h$ . The topography of a given highway is most economically described by the joint distribution of curvatures  $c_h$  and distances between transition points  $\Delta x_h$ . To illustrate this point, tables 1 and 2 and figure 2 give the successive curvatures and arc-length distances between transition points for a test section of highway used in our experiments.  $c_h$  appears usually to have zero mean and a unimodal distribution with range on the order  $\pm 0.2$  deg/ft, while  $\Delta x_h$  has a positive mean and unimodal distribution. While highway data are readily available, their topography does not appear to have been subjected to statistical analysis, so that little more can be said of the highway-curvature forcing function.

A second group of highway-produced disturbances is that due to sidewinds, irregular pavement, etc. These seem to be best described collectively as a random zero-mean addition to the vehicles' yaw rate. Beyond the fact of their existence we have little data to go on, but it seems plausible to assume that the total forcing function, which is the sum of many small incoherent components, can be described as a Gaussian disturbance with limited bandwidth and variance dependent on forward speed.

Highway surface friction is also important since in combination with tire mechanics it provides side forces for vehicle steering. In general we assume high  $\mu$  conditions providing a large linear range of steering response.

**VEHICLE DYNAMICS.**—As shown by Wohl (ref. 2) among others, a purely "kinematic" vehicle would have a linear transfer function  $K/s^2$  from steering wheel angle to lateral position on the highway. This can be considered to be generated in two tandem stages

- (1) a time-independent effect of steering angle on path traversed
- (2) the effect of forward speed which transforms path traversed into a time function

This ideal second-order response is actually obtained at very low speeds. At higher speeds a variety of secondary dynamic effects appear. These have been studied by Segel (ref. 8), Nordeen (ref. 9), Weir, Shortwell, and Johnson (ref. 10) and many other authors under the general heading of vehicle handling characteristics. Space does not permit full discussion of the many theories and data presented, but certain generalizations may be plausibly asserted to hold true for most passenger cars.

(1) At limiting low forward speeds ( $v_c = dx_c/dt \rightarrow 0$ , where  $v_c$  = vehicle forward speed), the curvature  $c_c$  of the car's path in space is proportional to steering wheel angle  $\theta_s$ . This is the ideal, Ackerman, steering response. Hence

$$c_c = \left( \frac{\delta \alpha_c}{\delta x_c} \right) = K_0 \theta_s \quad (v_c \rightarrow 0) \quad (1)$$

$K_0$  is typically about  $6.5 \times 10^{-3}$  deg/ft/deg. This point is illustrated by tables 3 and 4, which give the distribution of values of the ideal curvature-response gain  $K_0$  derived from data for 93 European cars tested by the British Consumer's Association in the period 1962 to 1967 and reported in *Motoring Which?* (ref. 11).

Equation (1) can be rewritten in the small-signal form

$$\left[ \frac{\partial c_c}{\partial \theta_s} \right]_{v \rightarrow 0} = \text{Const.} \quad (2)$$

TABLE 1.—TOPOGRAPHY OF BEAR CREEK ROAD,  
CONTRA COSTA COUNTY, CALIF.

| Segment                    | Length<br>(feet) | Horizontal<br>Curvature<br>(deg/feet) | Segment        | Length<br>(feet) |        | Segment | Length<br>(feet) |        |
|----------------------------|------------------|---------------------------------------|----------------|------------------|--------|---------|------------------|--------|
| 1                          | 302              | 0.000                                 | 21             | 1,476            | 0.000  | 51      | 627              | 0.000  |
| 2                          | 1,696            | +0.088                                | 22             | 782              | -0.064 | 52      | 1,129            | +0.088 |
| 3                          | 598              | 0.000                                 | 23             | 270              | 0.000  | 53      | 2,641            | 0.000  |
| 4                          | 859              | -0.088                                | 24             | 273              | +0.027 | 54      | 756              | -0.082 |
| 5                          | 2,003            | 0.000                                 | 25             | 272              | 0.000  | 55      | 55               | 0.000  |
| 6                          | 344              | -0.088                                | 26             | 876              | -0.042 |         |                  |        |
| 7                          | 510              | 0.000                                 | 27             | 626              | 0.000  |         |                  |        |
| 8                          | 639              | +0.088                                | 28             | 674              | -0.057 |         |                  |        |
| 9                          | 490              | 0.000                                 | 29             | 506              | 0.000  |         |                  |        |
| 10                         | 539              | -0.088                                | 30             | 653              | -0.088 |         |                  |        |
| 11                         | 387              | 0.000                                 | 31             | 404              | 0.000  |         |                  |        |
| 12                         | 1,118            | +0.112                                | 32             | 740              | +0.052 |         |                  |        |
| 13                         | 400              | 0.000                                 | 33             | 815              | 0.000  |         |                  |        |
| 14                         | 400              | -0.088                                | 34             | 284              | +0.088 |         |                  |        |
| 15                         | 524              | 0.000                                 | 35             | 400              | 0.000  |         |                  |        |
| 16                         | 465              | +0.029                                | 36             | 414              | -0.088 |         |                  |        |
| 17                         | 587              | 0.000                                 | 37             | 300              | 0.000  |         |                  |        |
| 18                         | 640              | -0.019                                | 38             | 615              | +0.088 |         |                  |        |
| 19                         | 2,820            | 0.000                                 | 39             | 400              | 0.000  |         |                  |        |
| 20                         | 1,027            | -0.028                                | 40             | 775              | -0.088 |         |                  |        |
| Total Length               |                  |                                       | 33,211 feet    |                  |        |         |                  |        |
| Mean Length of Segments    |                  |                                       | 736 feet       |                  |        |         |                  |        |
| Mean Curvature of Segments |                  |                                       | 0.01 degree/ft |                  |        |         |                  |        |

(2) Equation (2) continues to describe the small-signal low-frequency response of the vehicle throughout the working range of forward speed and curvature but the curvature-response gain parameter  $K(v_c, c_c)$  is a weak function both of forward speed  $v_c$  and of mean path curvature  $c_c$

$$\left[ \frac{\partial c_c}{\partial \theta_s} \right]_{v_c, c_c} = K(v_c, c_c) \quad (|v_c| > 0) \quad (3)$$

TABLE 2.—DISTRIBUTION OF LENGTH AND CURVATURE  
OF HIGHWAY SEGMENTS

| Length             | Number | Curvature                | Number |
|--------------------|--------|--------------------------|--------|
| 0 - 199 feet       | 0      | 0.125 - 0.175 deg/feet   | 0      |
| 200 - 399 feet     | 8      | 0.075 - 0.124 deg/feet   | 6      |
| 400 - 599 feet     | 14     | 0.025 - 0.074 deg/feet   | 3      |
| 600 - 799 feet     | 11     | -0.025 - 0.024 deg/feet  | 23     |
| 800 - 999 feet     | 3      | -0.075 - -0.026 deg/feet | 4      |
| 1000 - 1199 feet   | 3      | -0.125 - -0.076 deg/feet | 3      |
| 1200 - 1399 feet   | 0      | 0.175 - -0.126 deg/feet  | 0      |
| 1400 - 1599 feet   | 1      | Total Segments           | 44     |
| 1600 - 1799 feet   | 1      | Mean Curvature           | .0056  |
| 1800 - 1999 feet   | 0      | S.D.                     | .017   |
| 2000 and over feet | 3      | Variance                 | .003   |
| Total Segments     | 44     |                          |        |
| Mean Length        | 736 ft |                          |        |
| S.D.               | 167 ft |                          |        |

The curvature-response gain function has not been fully explored in the literature. Published data of Segel (ref. 8) on a 1953 Buick sedan and our own measurements on a 1965 Ford sedan test car (ref. 12) (fig. 3(a)), indicate that at constant  $c_c$  there is a 60 or 70 percent reduction in  $K$  over the normal speed range (0 to 70 mph). However, published data for certain high-performance vehicles such as the Lotus Elan tested by Motoring Which? (ref. 11) show very small percentage decrease of gain with speed (fig. 3(b)). Varying  $c_c$  with constant  $v_c$  is a less frequently performed test, and results here show quite variable patterns (ref. 11). However, subjectively better handling cars again show small percentage changes in  $K$  over the normal curvature range  $c = \mp 0.5$  deg/ft. Taking the available data together, we can reasonably infer that ideal car-handling qualities would call for completely linear curvature response, that is the small-signal path curvature response gain  $K$  should be invariant under changes of forward speed and of mean curvature. This response pattern can of course only be maintained up to some limiting value of lateral acceleration  $a_r$  generally between 0.6 to 0.9 g where

$$a_r = c_c \cdot v_c^2 \quad (4)$$

Above this point  $K$  begins to change rapidly either downwards (limiting understeer) or upwards (limiting oversteer) and even approximate linearity can no longer be assumed.

(3) The dynamic response of path curvature to steering wheel input is mechanically complicated but appears to be well approximated for frequencies of interest in normal steering by a second-order filter with breakpoint about 1.0 hertz. Here again there is a serious lack of data from which valid generalizations might be derived. Our own studies on a 1965 Ford

sedan test car (fig. 4) indicate that linear response is maintained up to perhaps 10 hertz with some high-order dynamics due mainly to tire and suspension effects. The main break-frequency around 1 hertz presumably varies with type of car, forward speed, and path curvature, but in an unknown manner.

Yaw angle and lateral position displacement responses can be readily derived geometrically from the path-curvature responses outlined above and are respectively proportional to  $v_c$  and  $v_c^2$ .

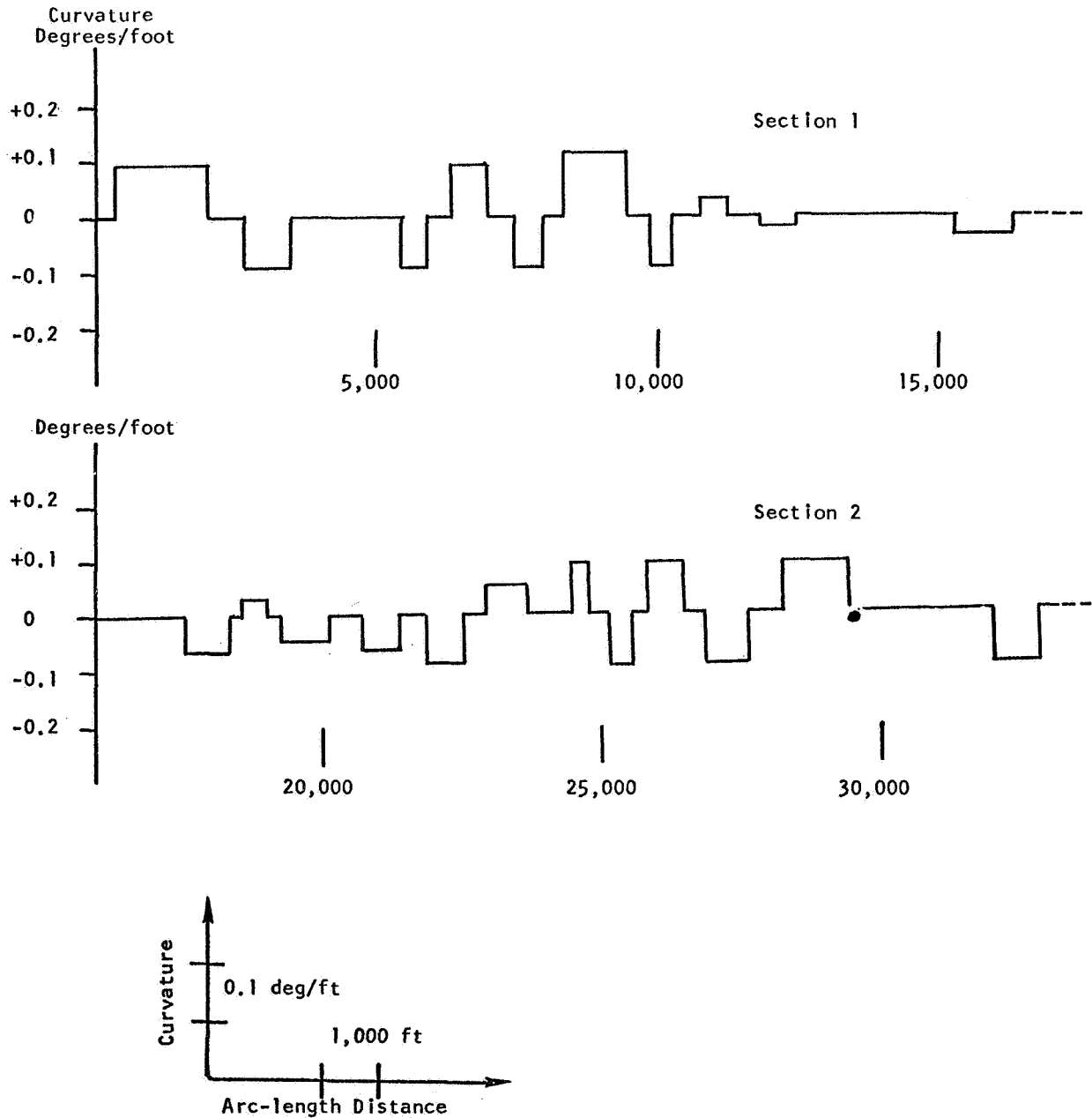


Figure 2(a). — The highway as forcing function. A 33 000-foot section of Bear Creek Road, Contra Costa County, Calif.

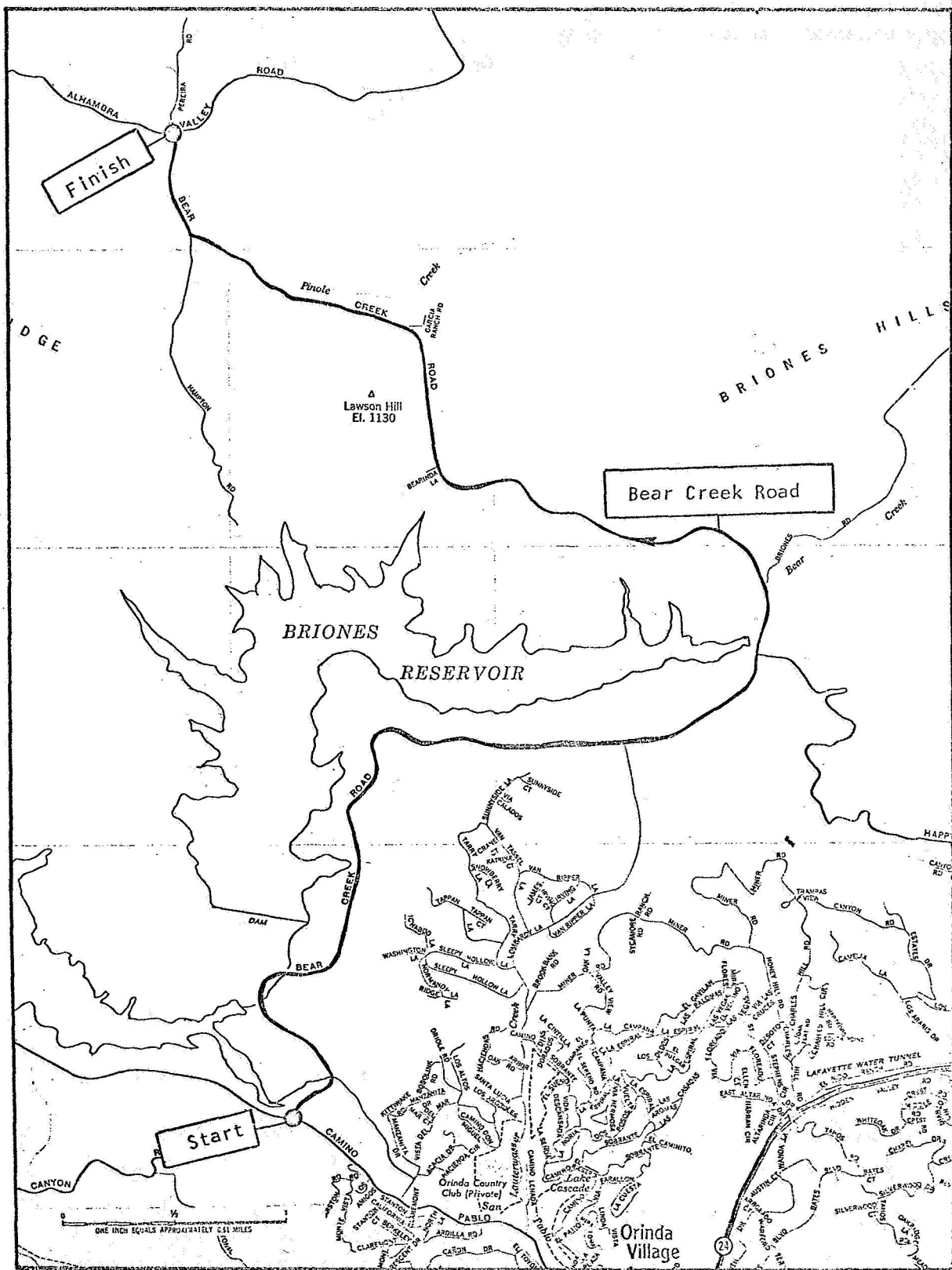


Figure 2(b). —Plan view of Bear Creek Road.

TABLE 3.—SMALL-SIGNAL CURVATURE-RESPONSE GAIN OF 100 EUROPEAN CARS  
TESTED BY CONSUMER'S ASSOCIATION, LONDON, 1962-7

| Gain<br>(deg/ft/deg x 1000) | Frequency<br>(Percentage) |
|-----------------------------|---------------------------|
| 3 - 3.9                     | 0                         |
| 4 - 4.9                     | 4                         |
| 5 - 5.9                     | 29                        |
| 6 - 6.9                     | 22                        |
| 7 - 7.9                     | 17                        |
| 8 - 8.9                     | 10                        |
| 9 - 9.9                     | 0                         |
| 10 and over                 |                           |
| Mean Gain                   | 0.0065 deg/ft/deg         |
| S.D.                        | 0.0011 deg/ft/deg         |

TABLE 4.—PERCENT REDUCTION OF CURVATURE-RESPONSE GAIN AT 0.25 g  
LATERAL ACCELERATION AT TWO CONSTANT CURVATURES FOR 51 CARS  
MEASURED BY CONSUMER'S ASSOCIATION, LONDON

| Percent<br>Reduction      | Frequency<br>(Percentage)<br>Curvature 0.76 deg/ft<br>(75 ft radius) | Frequency<br>(Percentage)<br>Curvature 0.155 deg/ft<br>(350 ft radius) |
|---------------------------|----------------------------------------------------------------------|------------------------------------------------------------------------|
| 0 - 9                     | 8                                                                    | 0                                                                      |
| 10 - 19                   | 36                                                                   | 0                                                                      |
| 20 - 29                   | 54                                                                   | 4                                                                      |
| 30 - 39                   | 4                                                                    | 8                                                                      |
| 40 - 49                   | 0                                                                    | 19                                                                     |
| 50 - 59                   |                                                                      | 42                                                                     |
| 60 - 69                   |                                                                      | 27                                                                     |
| 70 - 79                   |                                                                      | 0                                                                      |
|                           | n = 25                                                               | n = 26                                                                 |
| Mean Percent<br>Reduction | 20.3                                                                 | 53.0                                                                   |

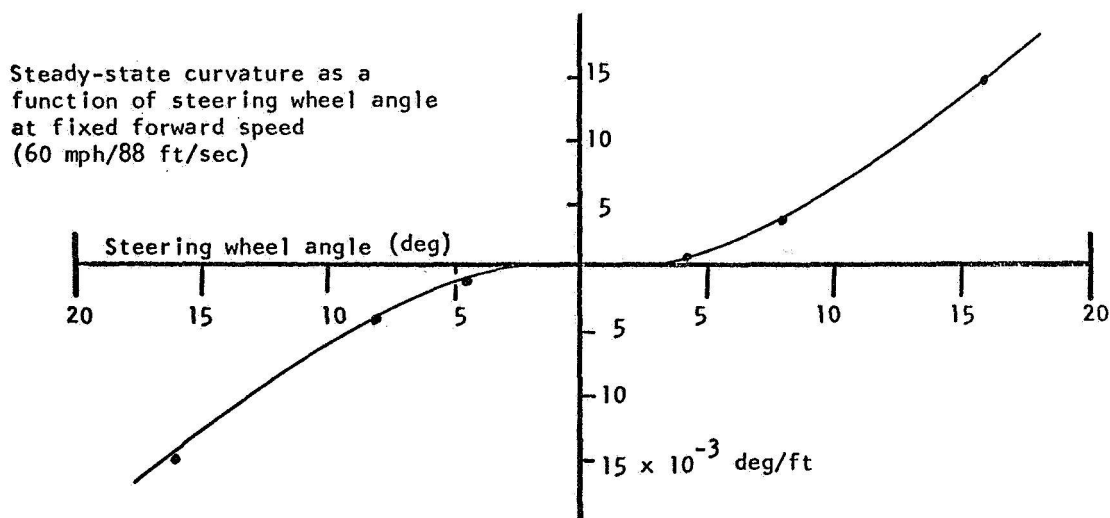
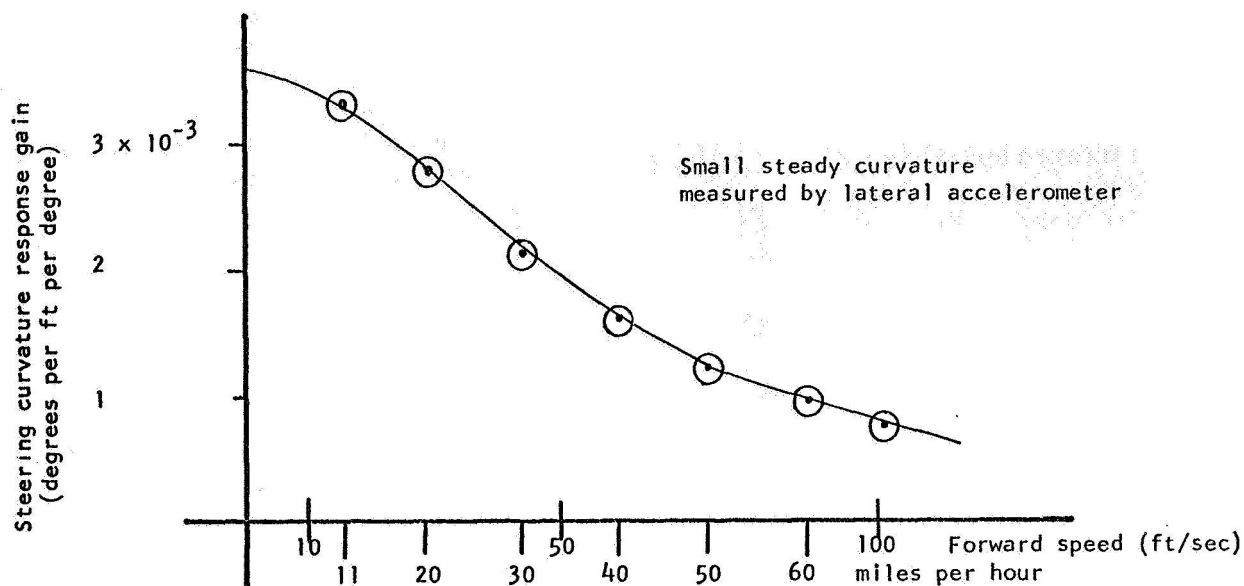


Figure 3(a).—Measured curvature response of a 1965 Ford sedan.

(4) Torque approximately proportional to angular displacement is needed to deflect the steering wheel from its central or straight-ahead position. Numerical values on the order of 0.1 lb/deg are typically recorded for non-power-steering cars (ref. 11).

### A THREE-LEVEL DRIVER MODEL

**LEVEL 1: OPEN-LOOP CURVATURE CONTROL USING PREVIEW.**—Since highways tend to show long sections of constant curvature connected by well-defined transitions to new curvature values, and since the vehicle has approximately constant gain from steering wheel angle to path curvature, a minimal control policy could be implemented as follows: The

driver scans the road ahead and detects points at which the highway changes curvature. He estimates the new curvature while approaching the transition point, translates this mentally into a required new steering wheel setting, computes the torque to be applied, and executes the required change at about the time the car comes abreast of the transition point.

Given a car with ideal steering characteristics this policy could be executed independently of forward speed, thus decoupling the steering and speed control subsystems, a major advantage given the normal requirement for frequent speed-variations due to traffic, gradients etc. However, even in this case the success of the policy would obviously be limited by the precision of the driver's curvature estimation. No data are available from actual highway situations to estimate this parameter, but laboratory studies by Pollack (ref. 13) indicate about 2-bit precision. While this figure may well be subject to improvement under the kind of extended practice with knowledge of results that is obtained in normal driving, we should probably assume that less than 3.0 bits of absolute curvature information would be available,

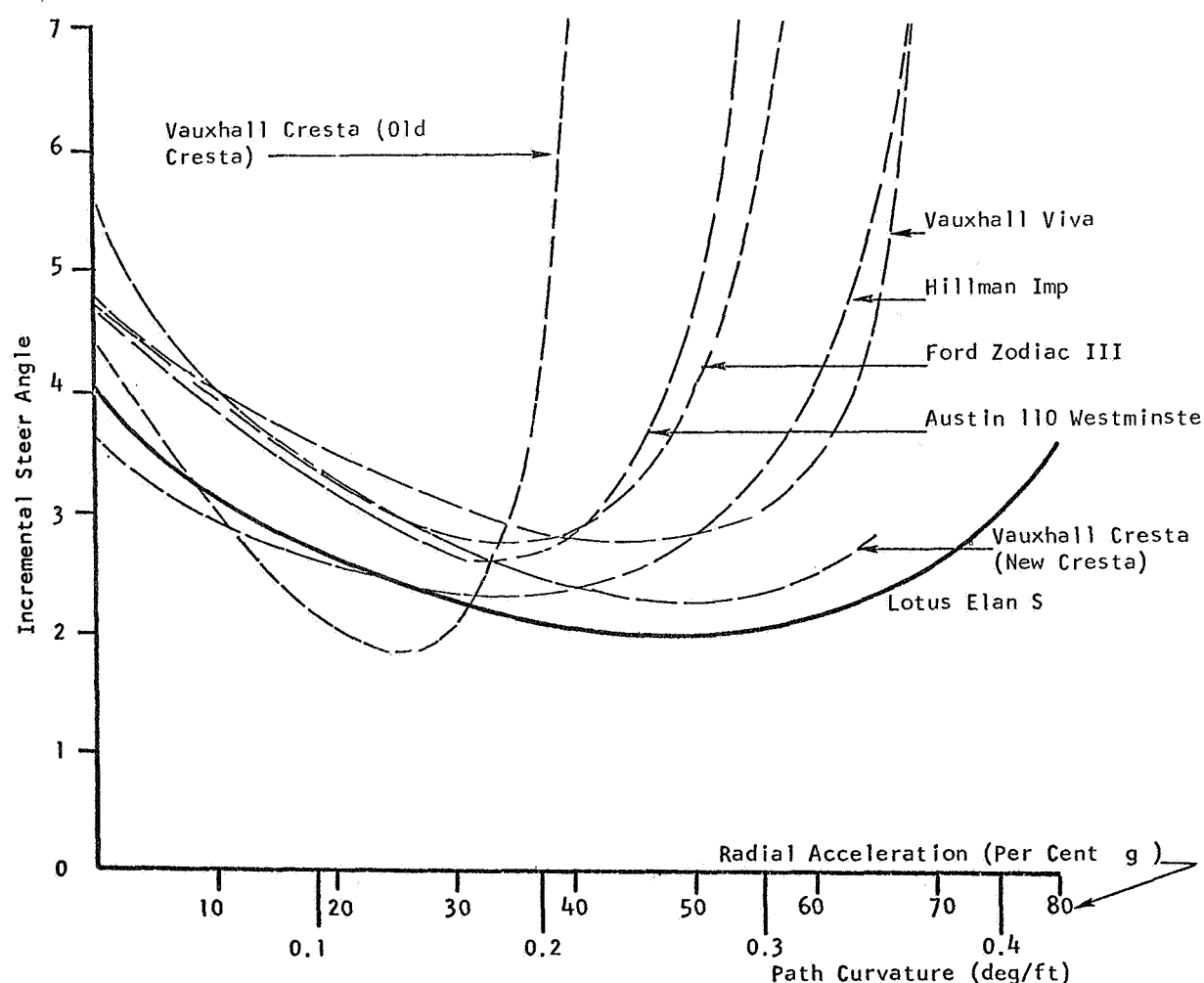


Figure 3(b).—Incremental steer angle per unit increase in curvature, measured by Consumers Association, London. Constant speed 40 mph, inward spiral course.

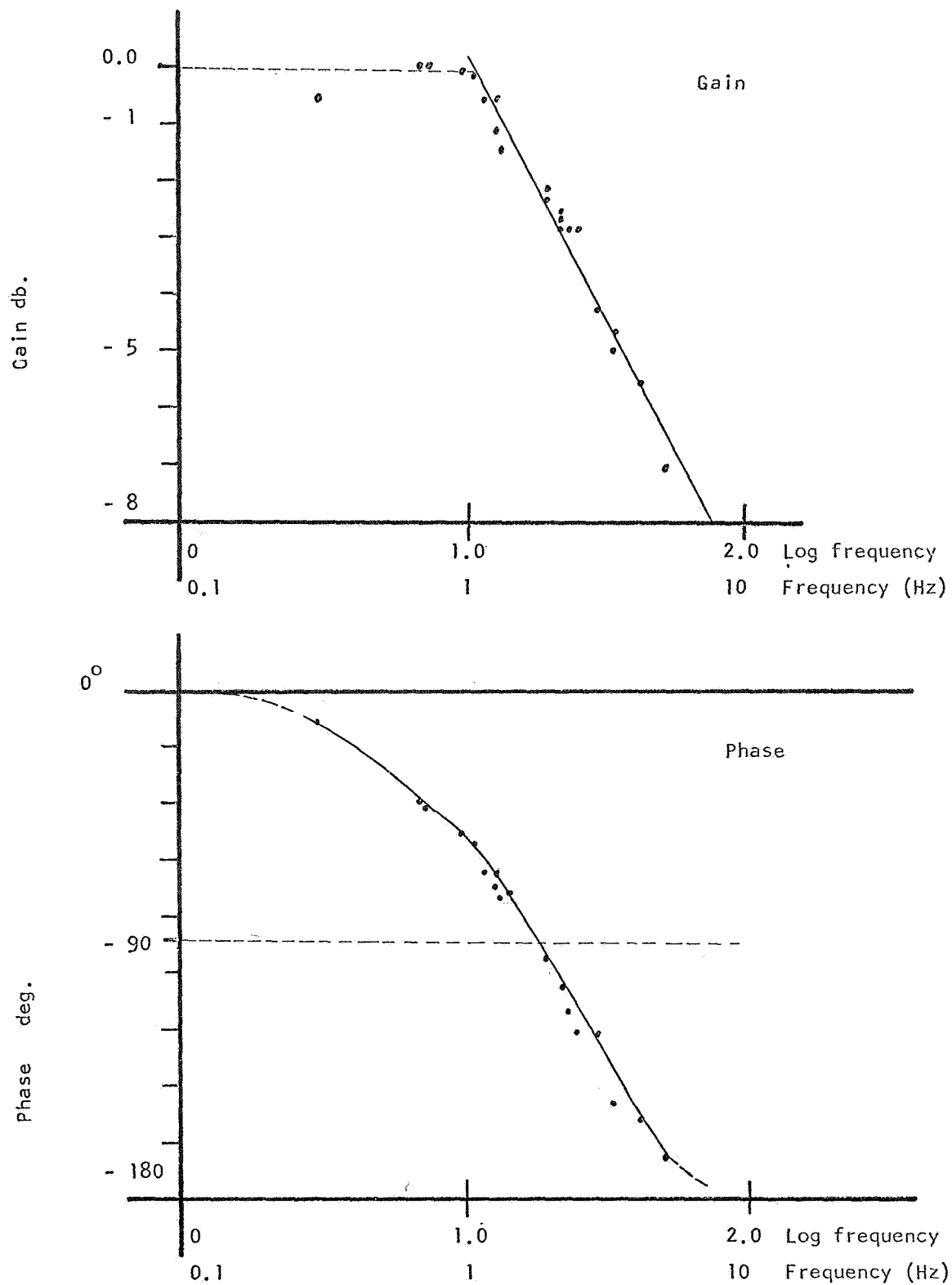


Figure 4.—Measured yaw-rate and path curvature frequency-response of a 1965 Ford sedan (ref. 12).

including the 1-bit sign. Since information would undoubtedly be lost in computing the required steering adjustment, this open-loop control model must be assumed capable of producing a vehicle path curvature correlated less than  $r=0.90$  with highway curvature. Since lateral-position error is a double time-integral function of curvature error, this control mode is clearly incapable of fully meeting the performance criteria outlined above. However, its adoption would substantially reduce the load placed on the closed-loop modes described below.

Since this is a strictly open-loop policy it requires the output member to be in some sense calibrated to permit setting at a desired value. This appears to be accomplished by combined use of a central (zero-curvature) straight-ahead reference position obtained (in an ideal car and to good approximation in most real cars) by applying zero torque to the steering wheel, and an approximately linear torque/displacement curve. The driver may be supposed to apply a known torque to the wheel in the expectation that this will result in a proportional path curvature.

We have little direct support for this conjecture, but we have found it essential to provide our simulator with realistic feel including a central reference point. This is achieved by the use of opposed springs providing force proportional to angular displacement from the central straight-ahead position. Numerous steering wheel torque measurements reported in *Motoring Which?* (ref. 11) also support this view.

A suitable flow-chart for executing this hypothetical first-level control policy is shown in figure 5. Assuming the component processes to be computed on a timescale of fractions of a second, the corresponding continuous transfer-function driver model would be a simple proportionality with added noise, the cutoff frequency being considerably higher than that needed to respond adequately to highway variations (fig. 6).

The only direct data we can currently adduce to support this hypothesis are recordings of eye fixations (fig. 7) which show that the driver's attention is certainly placed where it would be best able to acquire the necessary data, viz. in the middle distance and deflected in the direction of curves currently being negotiated. Further detailed study of eye-fixation and steering response patterns will be needed, and we also plan to analyze the pattern of driver response to curvature transitions in search of the predicted pattern of advance preparation and transient error.

LEVEL 2: CLOSED-LOOP CONTROL OF RELATIVE HEADING AND/OR RELATIVE PATH ANGLE AND/OR LATERAL POSITION RATE.—Assuming that the above open-loop policy can only achieve partial success, there will be residual curvature error, the difference between highway and car-path curvature. Let this be  $c_e$ , where

$$c_e = c_h - c_c \quad (5)$$

At a given forward speed  $v_c$  this results geometrically in a proportional time rate of increase of the angle between car-path heading and highway direction, the relative path angle  $\alpha_{rel} = \alpha_h - \alpha_c$

$$\frac{d(\alpha_{rel})}{dt} = \frac{d\alpha_h}{dx_c} \cdot \frac{dx_c}{dt} = v_c \cdot c_e \quad (6)$$

$$\frac{d^2 y_c}{dt^2} = \frac{d(\alpha_{rel})}{dt} \cdot \frac{dx_c}{dt} = v_c^2 \cdot c_e \quad (7)$$

where  $y_c$  = lateral position of the car relative to the highway centerline.

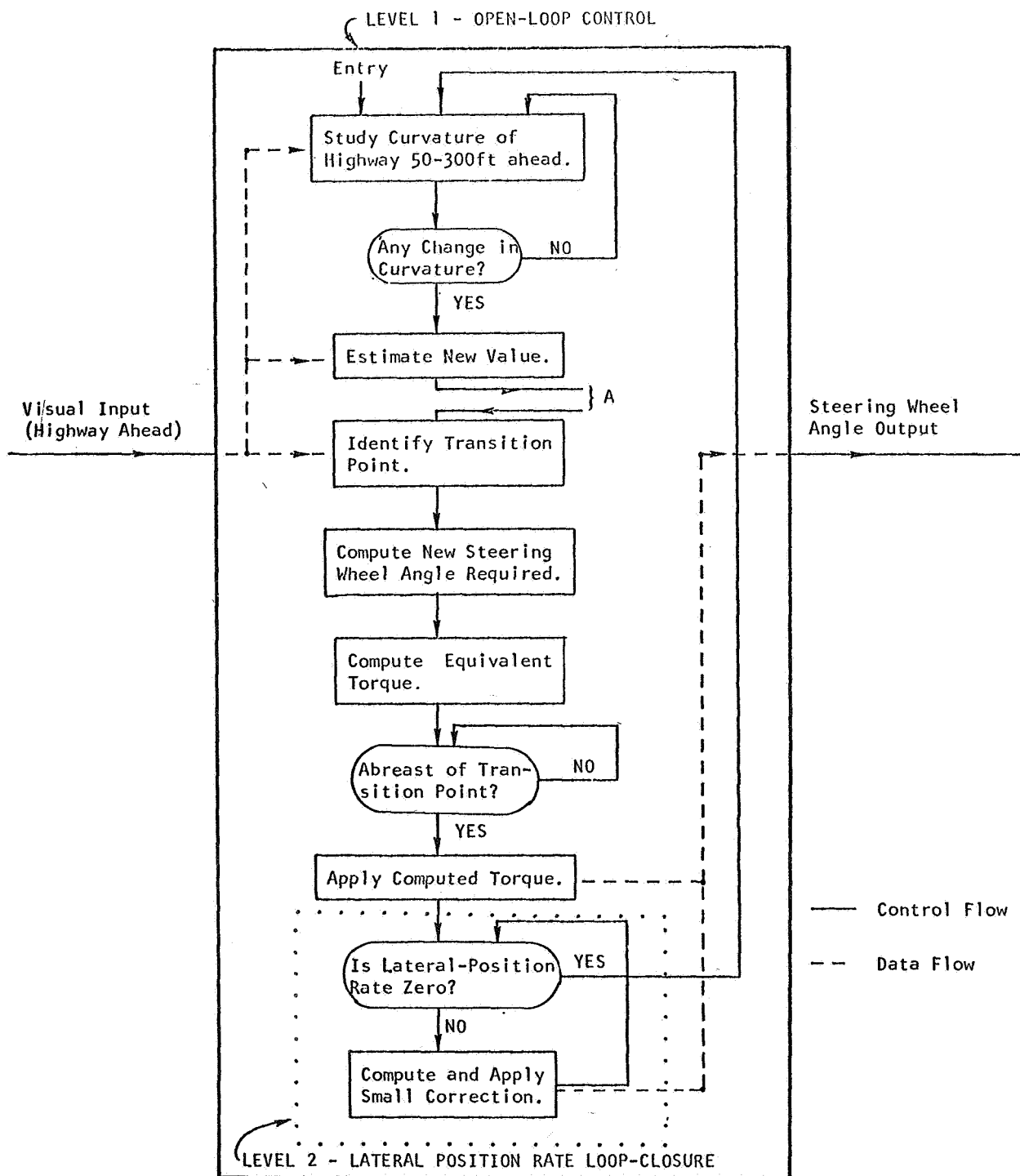


Figure 5. —Flow chart for open-loop preview control of path curvature in highway steering. (A speed-control subroutine may be interpolated at point A.)

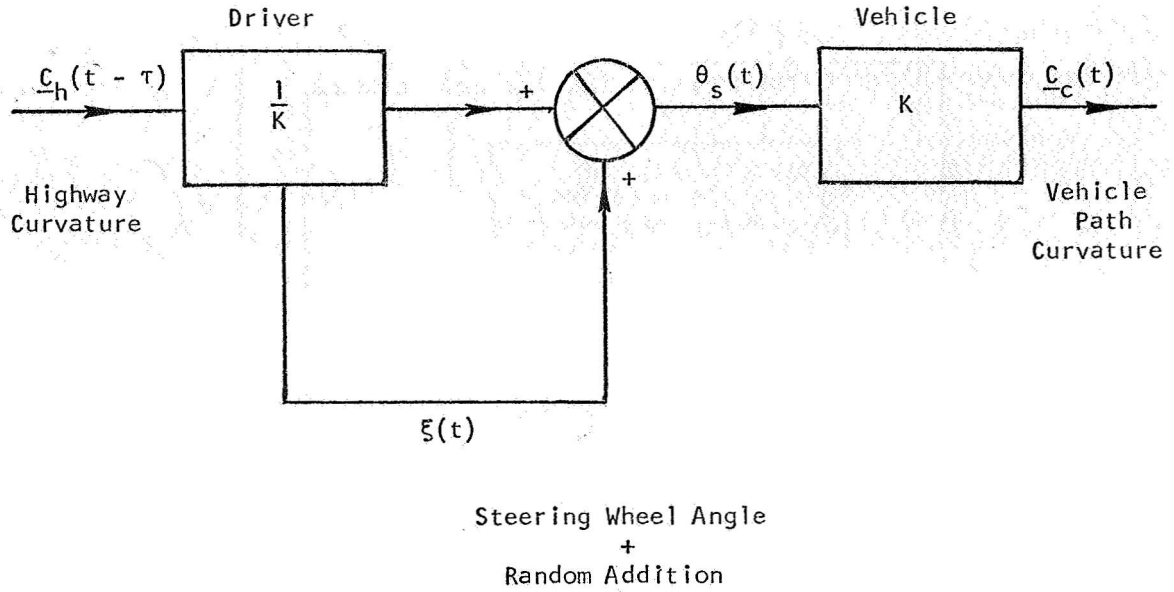


Figure 6.—Equivalent flow diagram for open-loop preview control of path curvature routine shown in figure 5.

With an idealized vehicle as described on the previous page this yields the fundamental equation of vehicle steering

$$\frac{d^2 y_c}{dt^2} = v_c^2 \cdot K \cdot \theta_s \quad (8)$$

The steering task calls for the car's lateral position  $y_c$  to be kept equal to a desired or reference lateral position  $y_r$ , normally generated internally by the driver. Direct continuous negative feedback utilizing an estimate of  $y_e$  of course results in an unstable system, since the idealized open-loop response is

$$\frac{y_e}{\theta_s} = \frac{v_c^2 K}{s^2} \quad (9)$$

In the nonideal case there is also a tandem second-order lag above 1.0 hertz, making the position worse.

However, given the highly nonlinear cost-function associated with  $y_e$ , the reduction in low-frequency error power achieved by the open-loop curvature-control policy and the moderate high-frequency power normally present in the two forcing functions, there appears to be no need for a tight loop operating directly on  $y_e$ . We postulate a tight loop (high repetition rate or sampling frequency) operating on lateral position rate or on one of the intermediate variables which are by geometry proportional to it. These are relative path angle  $\alpha_{rel} = \alpha_h - \alpha_c$  and the relative heading  $\psi_{rel} = \alpha_h - \psi_c$  where  $\psi_c$  equals instantaneous angular heading of the car to a first approximation  $\alpha_c = \psi_c$ . At low curvatures and low lateral accelerations relative headings  $\psi_{rel}$  can presumably be estimated with good precision (perhaps 3 bits) by direct comparison of reference points or lines on the vehicle with distant points on the

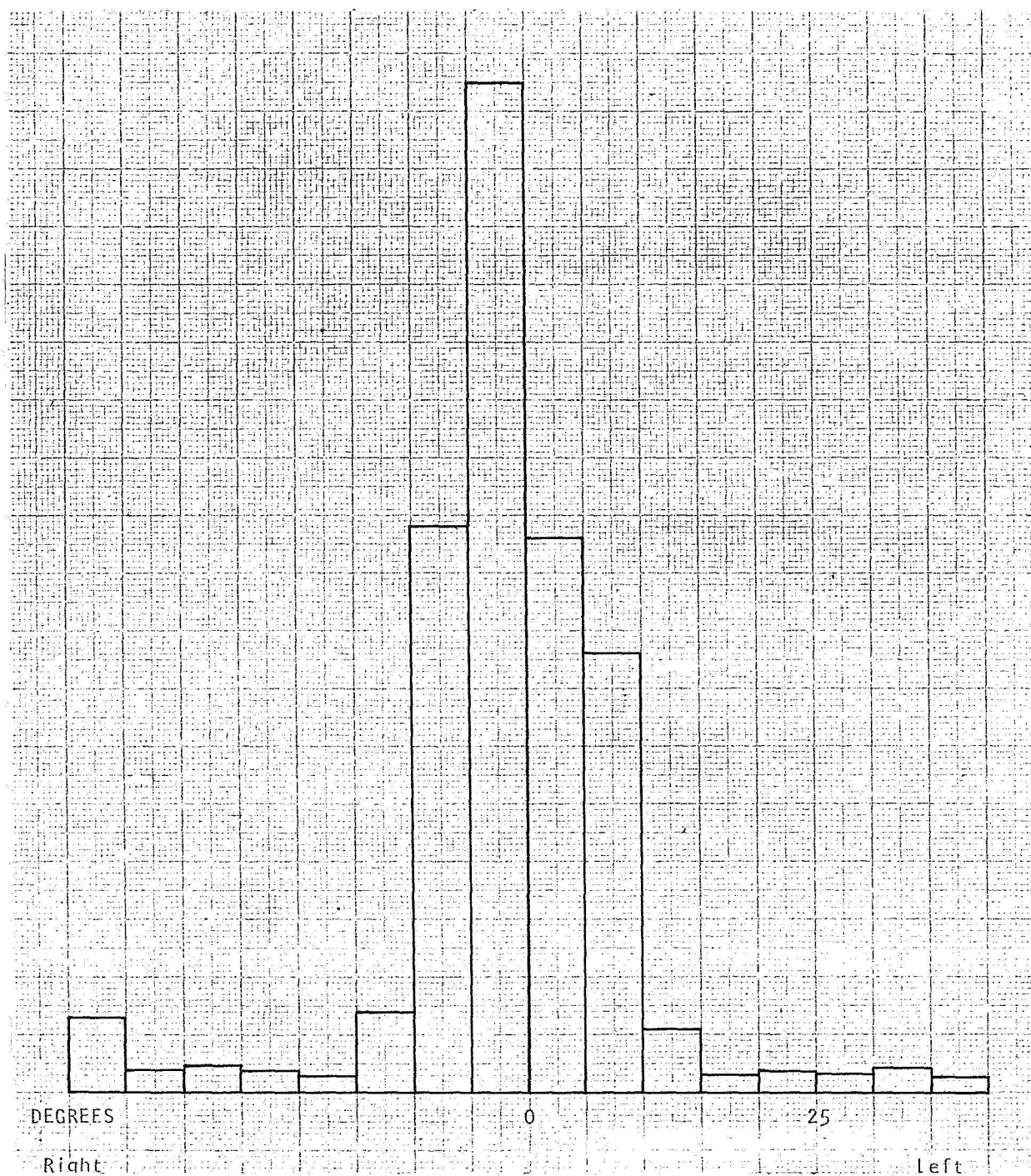


Figure 7.—Horizontal distribution of eye fixations in steering.  
A low-curvature country highway (Interstate 680 between  
Diablo and Walnut Creek, Contra Costa County).

highway or in the surrounding scenery. At high lateral accelerations vehicle sideslip complicates this picture, and  $\psi_{rel}$  ceases to be proportional to  $dy_c/dt$ . The angle  $\alpha_{rel}$  itself would then be better estimated directly. This can be done by using the "center of expansion" of the visual field as described by Gibson and Crooks (ref. 14) and Biggs (ref. 15).

However the estimate is obtained, the level 2 loop would be closed by computing and applying negatively proportional increments to the steering wheel angle initially set by the open-loop curvature policy. A sampling frequency on the order of 1 to 2 increments per second would normally suffice.

The functional importance of relative heading estimation has been underlined by our experience with a fixed-based simulator, in which we have found it essential to provide directional cues in the visual field. In their absence subjects are simply unable to maintain normal control, while in their presence most subjects will proceed to "drive" in a normal manner. We have no direct data on the use of center-of-expansion as distinct from relative-heading inputs.

The above policy breaks down on curves, where relative heading can no longer be directly estimated due to lack of highway-related directional cues. Center-of-expansion judgments are also disturbed by the lateral (angular) velocity of all points in the visual field due to the vehicle's rotation. While it is possible that the latter cue might yield enough data to implement the policy described above at a lower level of accuracy (higher threshold), a third more plausible alternative exists in lateral-position rate, which can still be estimated from "shear" velocity at the boundary between hood and pavement in the visual field. This is highly correlated with relative path angle and might be expected to replace it when the latter fails. If this actually happens higher lateral-position error would result on curves than on straightaways due to the probably lower precision of first derivative as compared with direct estimates of variables. Whichever of these three possible second-level policies is feasible at any particular time, the very simple flow-chart of figure 8(a) would suffice to implement it. The corresponding block diagram is shown in figure 8(b).

Since the driver devotes attention to other than steering tasks, this particular loop will probably not be cycling continuously, but may be entered after variable time spent in other processing. This will have the effect of reducing the equivalent low-pass filter break frequency, so that we would expect the closed-loop performance to show time-variable amounts of high-frequency error power, depending on what other tasks are being time shared with steering (refs. 16 and 17).

We have obtained evidence for a level 2 loop of this type in a test-car steering experiment using a quasi-random forcing function injected between the driver's output and the actual steering wheel. This was implemented by means of our "double steering wheel" technique that simulates random side-thrust applied to the car. The lateral-position error was recorded by means of a TV camera error sensor. Preliminary results show that the driver introduces slightly less than the theoretical 6 dB/octave increase in gain below about 0.5 hertz, with a flat response above this frequency (fig. 9). This result is consistent with a control policy based on relative path angle or its equivalent, together with a driver-produced first-order filter with break frequency about 0.5 hertz. This is not inconsistent with a 1-second sampling rate, but we have no direct evidence either of the occurrence of sampling or of its frequency. However, further analysis of the data should reveal details of the driver's output and hence permit inferences as to his sampling activities.

**LEVEL 3: CLOSED-LOOP CONTROL OF LATERAL-POSITION.** —Since the precision of estimation of relative heading, relative path-angle or lateral-position rate are all limited to 3 bits per observation and will thus have a definite threshold, the second-level control

policy described above will fail to precisely null out relative-heading errors, though it will considerably reduce their amplitude and hence restrict the first derivative of lateral position to low levels, in other words act as a low-pass filter. Given initially zero lateral-position error  $y_e$ , only minor infrequent corrective action will suffice to maintain lateral position at its reference value  $y_r$ .

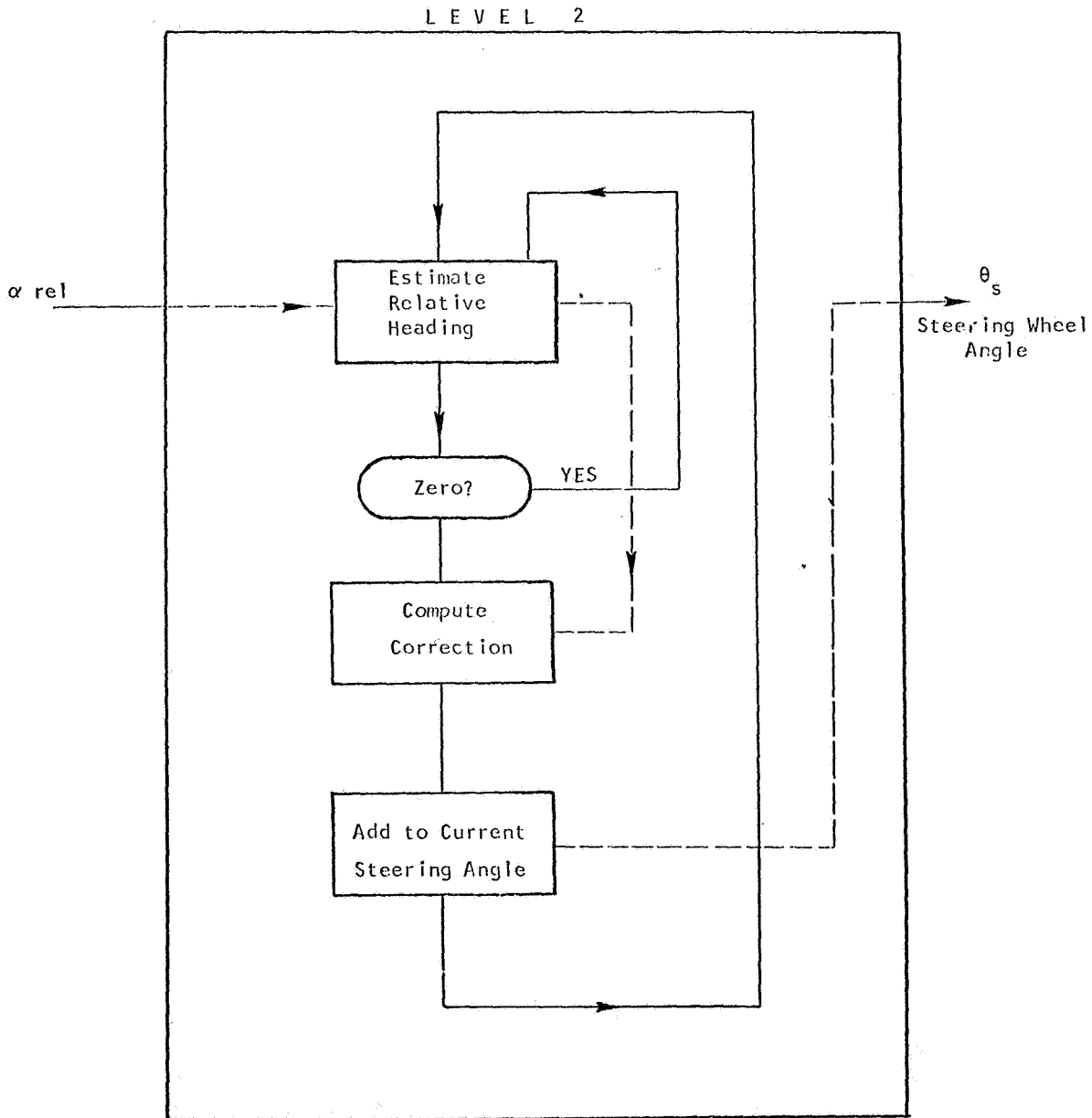


Figure 8(a). — Flow chart for level 2—Closed-loop compensatory regulation of relative heading.

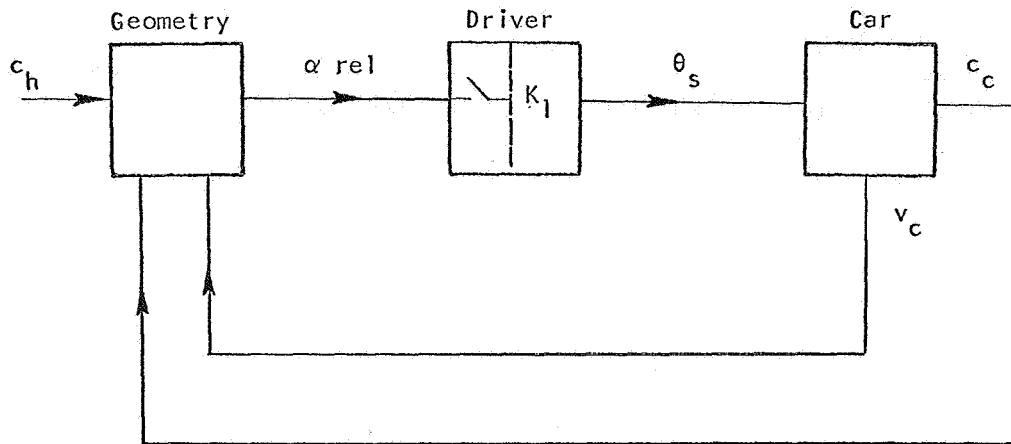


Figure 8(b). —Block diagram corresponding to figure 8(a).

A third-level policy to achieve this calls for direct estimation of lateral position  $y_c$  and its closed-loop adjustment to whatever reference value  $y_r$  is desired by the driver. As shown previously, this necessarily creates a second-order or higher order control system, but the low amplitude and low frequency of the residual forcing function applied to it, together with a relatively high error tolerance means that feedback gain can be kept low, for a low frequency of transient oscillation; a low sampling-rate will also suffice. Nevertheless we expect steady low-level oscillation to occur. This analysis is consistent with our observation of a low frequency periodicity in the 0.05 to 0.1 hertz region both in records of straight-highway simulator driving, and in actual studies of steering behavior on freeways.

To summarize the foregoing discussion, a model of the driver's steering behavior based on known parameters and constraints has been proposed viable for all normal highways and traffic conditions. It assumes a three-level hierarchy of control policies that reduce the difference between desired and actual path of the car, the former being defined jointly by the topography of the highway and the driver's preference for lateral positioning.

(1) Curvature of the highway ahead is estimated and the vehicle's path-curvature adjusted to conform to it by proportional change of steering wheel angle. This assumes that the driver "knows" the steady-state gain parameter relating steering wheel angle to path curvature.

(2) Residual error in car heading (or path angle) relative to the highway is nulled out by a negative proportional feedback with medium break frequency creating a first-order heading-control system and stabilizing the first derivative of lateral position.

(3) Lateral-position drift due to residual (subthreshold) discrepancy between highway direction and car path angle is nulled out by proportional feedback with low break frequency, creating a second-order lateral-position control system. This produces transient low-amplitude oscillation at frequencies on the order of 0.05 hertz.

It will be noted that no use has been made of lateral-acceleration data available from the driver's vestibular system and from the "seat of his pants." This is because in a well-behaved car on good surfaces lateral acceleration is dependent jointly on steering wheel angle and forward speed, both known quantities. Hence under these circumstances the vestibular information is redundant and could only serve to verify the occurrence of vehicle motions already commanded by the driver via the steering wheel. However in skids and slides the car's motion will no longer be exactly predictable from a knowledge of steering-wheel angle, so that vestibular data will presumably become useful in such cases. It will

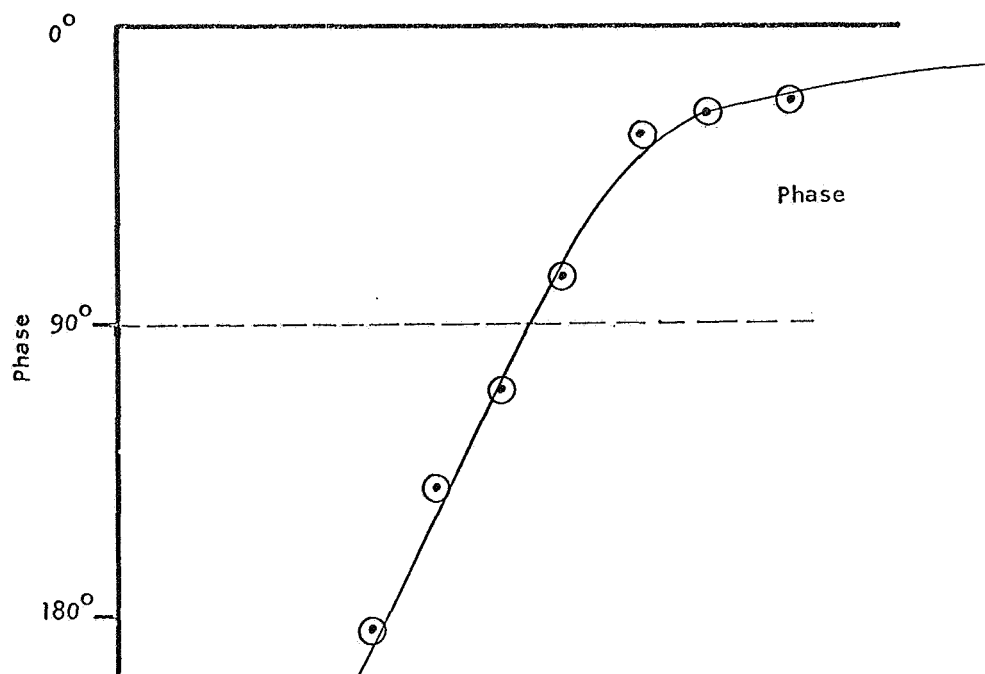
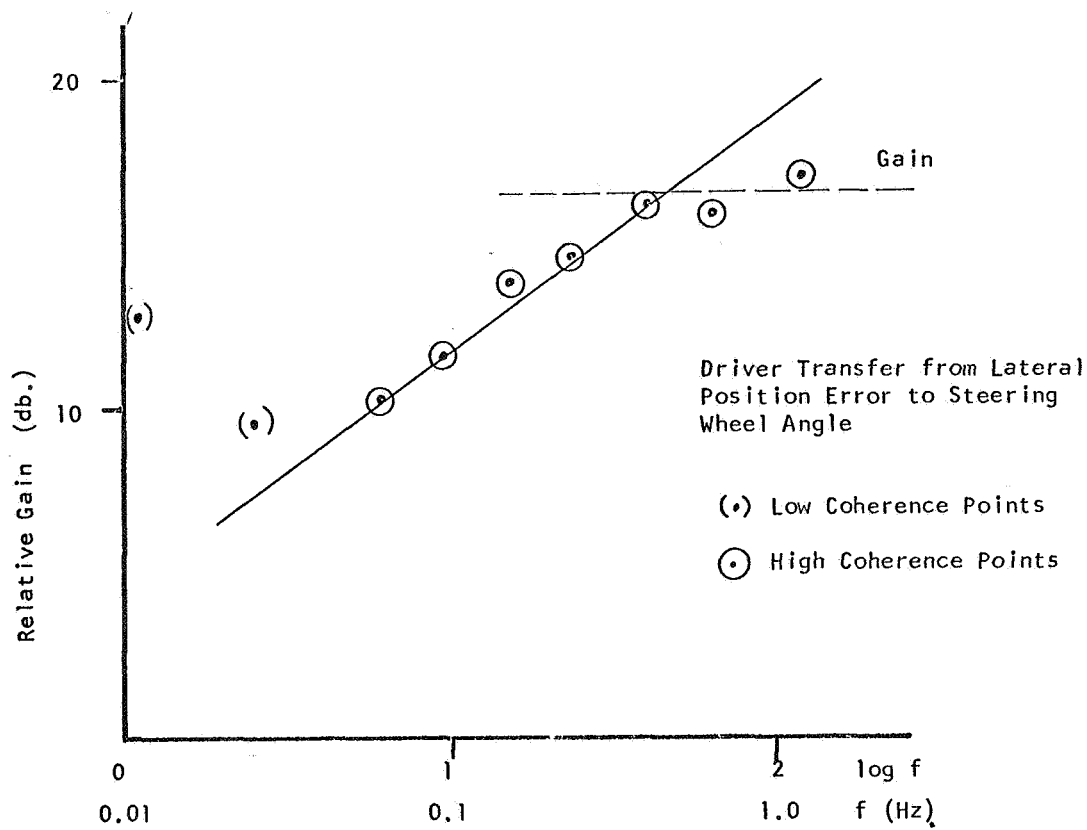


Figure 9.—Results of one subject in straight ahead driving with quasi-random forcing function introduced using the double steering wheel. 40 mph. S.T.I. forcing function.

also be noted that except for level 2 operating on curves all these policies call for proportional response, and none of them call for good high-frequency response. Thus the driver's response lag appears to be unimportant. A key question appears to be what are the actual and feasible minimum repetition rates (or sampling frequencies) at each level. While this certainly depends on the exact forcing function being applied at a particular time and on the various thresholds, it seems that level 1 and level 2 could operate with about a 1-1 ratio and a scan rate around 1 per second while level 3 would require perhaps 1/5 the scan rate. Data of Senders et al. are relevant here (ref. 18).

A suitable flowchart to implement all the levels together is shown in figure 10(a), and the corresponding sampled-data control system in figure 10(b). No claim is made that this model will fit all drivers all the time, but we believe it to be a realistic basis for further research aimed at estimating parameters and detailing driver response to different environments. We also feel that it may provide a useful framework for analyzing skill acquisition, since the levels could naturally be implemented in reverse order (3, 2, 1) with progressive improvement in stability, precision, and high-frequency response. At extremely high skill levels such as exhibited by racing drivers we would expect the precision of level 1 control to be such that relatively little error remains for the lower levels to remove. This might perhaps indicate the need for a level 0, using purely preprogrammed steering outputs whose timing alone would be adjusted to match the highway curvature.

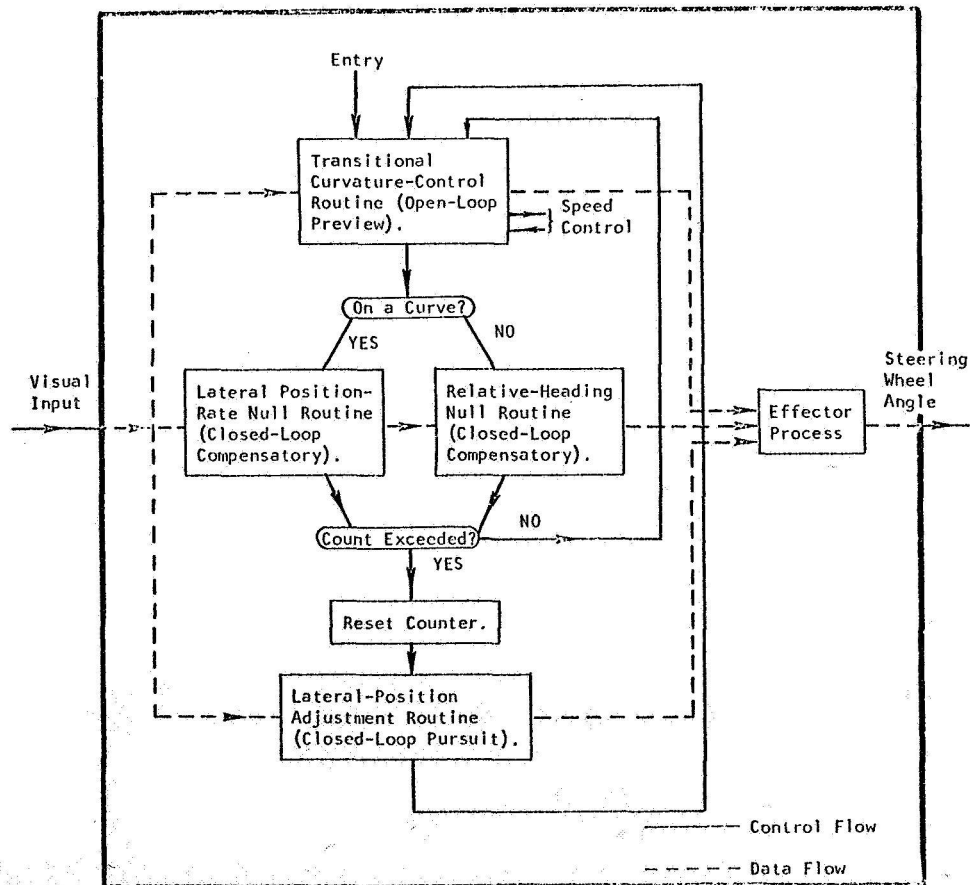


Figure 10(a).—Overall flow chart for driver information processing in vehicle steering.

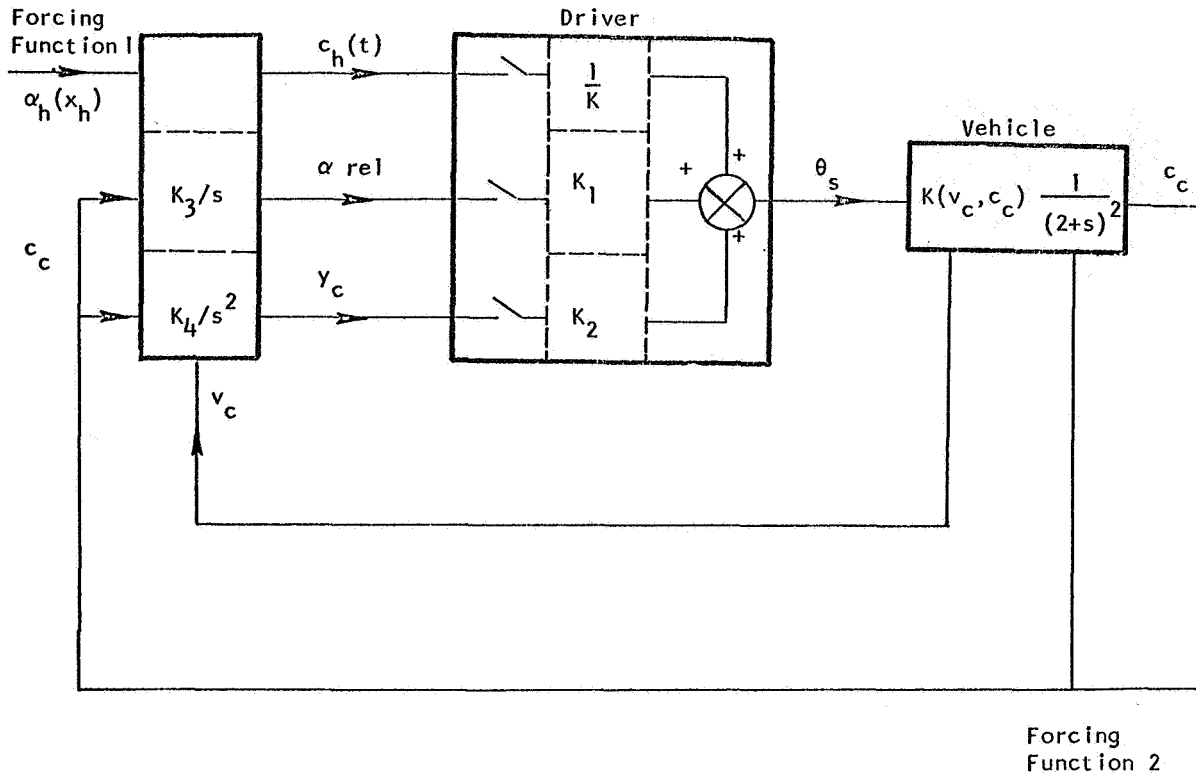


Figure 10(b).—Block-diagram equivalent of multilevel control policy shown in figure 10(a).

#### SPEED CONTROL IN RELATION TO STEERING

While this paper is primarily concerned with steering, the model presented above provides a natural framework for modeling the interaction between steering and speed control necessitated by the fact that steering fails on curves for which lateral acceleration  $a_r$  exceeds some threshold value  $a_{r\max}$ . Since speed control is implemented with long vehicle lags (ref. 10), the constraint on  $v_c$  implied by the condition

$$a_r = c_c \cdot v_c^2 \leq a_{r\max} \quad (10)$$

can only be met by feedforward control.

Since our level 1 already calls for estimates of highway curvature  $c_h$ , and since it is known that the vehicle's path curvature  $c_c$  cannot differ much from  $c_h$ , the necessary instantaneous  $v_{c\max}$  can be readily computed by solving the equation

$$v_{c\max} \leq (1/c_h)^{1/2} \quad (11)$$

While it is unrealistic to suppose that the driver solves inequality (11) on-line, he may readily use values of  $v_{c\max}$  "tabulated" against highway curvatures  $c_h$ , since only at most about 8 values of  $c_h$  can be discriminated. Clearly it is always possible to apply a safety factor by reducing the computed  $v_{c\max}$  by any desired amount. A suitable subroutine to implement this requirement is shown in figure 11 and may be inserted at point A in figures 5 and 10(a).

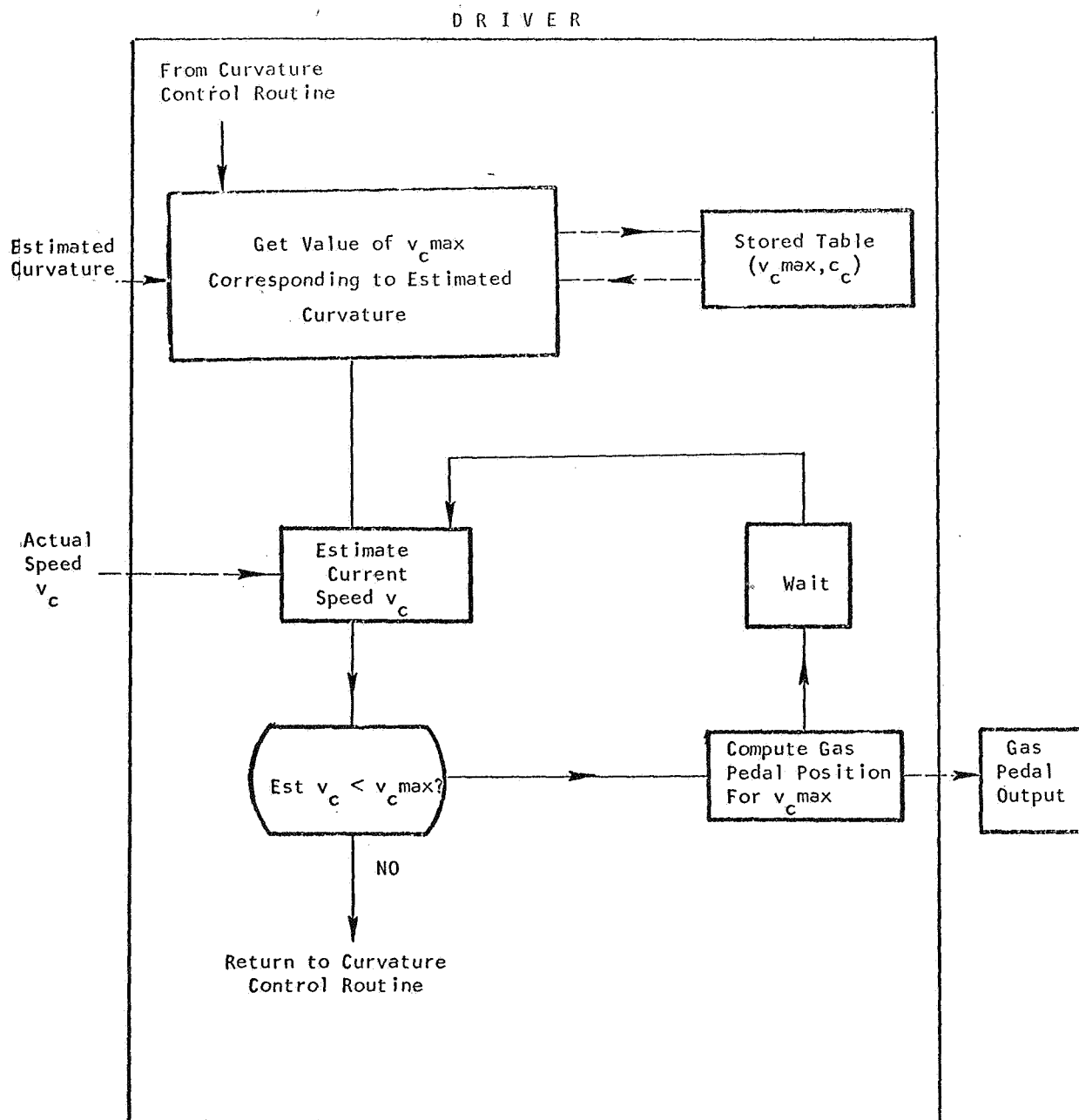


Figure 11.—Flow chart for speed-control adjustment routine to meet maximum lateral acceleration constraint.

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